Kondo Effect and Non-Fermi Liquid Behavior in Dirac Materials

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Collaborators

Kondo Effect in Dirac Materials

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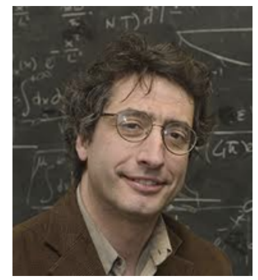

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Effect of Spin-orbit coupling on impurity bound states in superconductors

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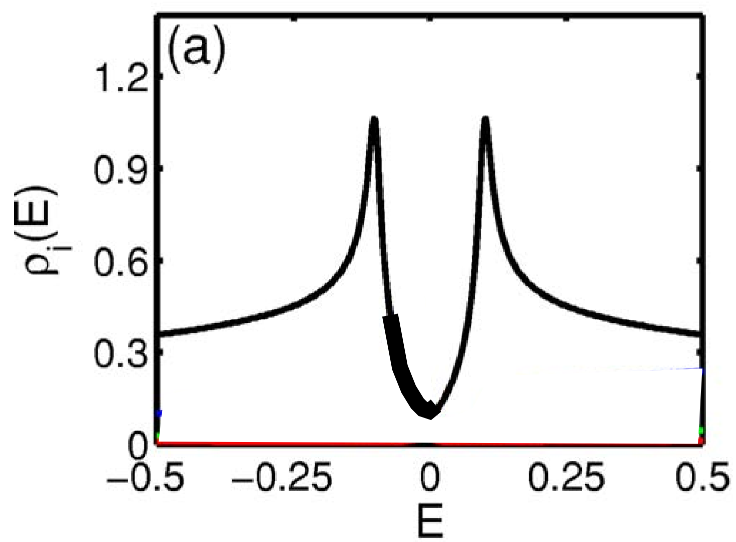
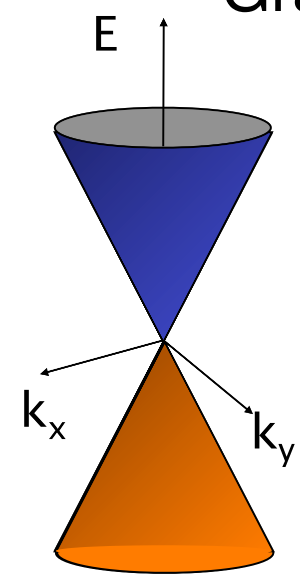
Dirac Materials

2D Dirac Materials

d-wave superconductors

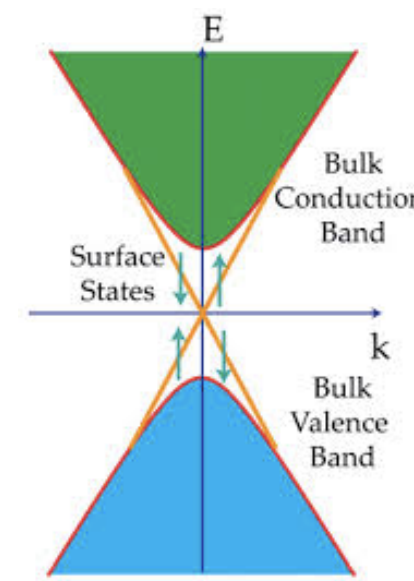


Graphene



$$\mathcal{H} = v_F \mathbf{p} \cdot \sigma$$

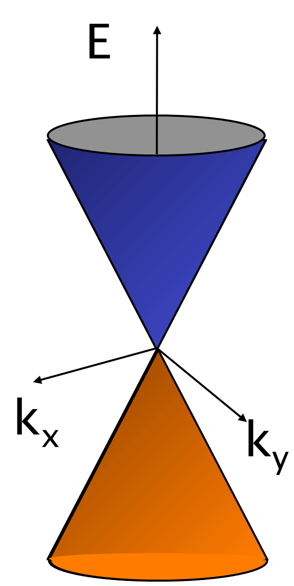
3D TI's surfaces

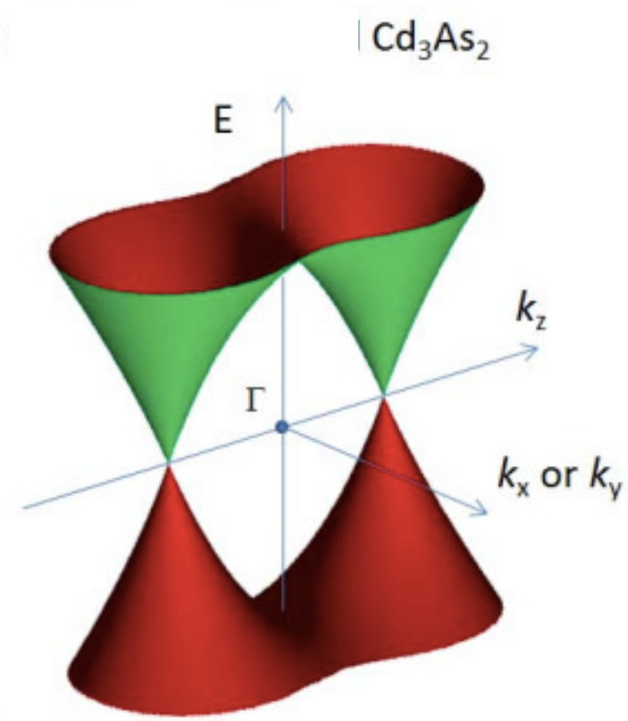


3D Dirac Materials

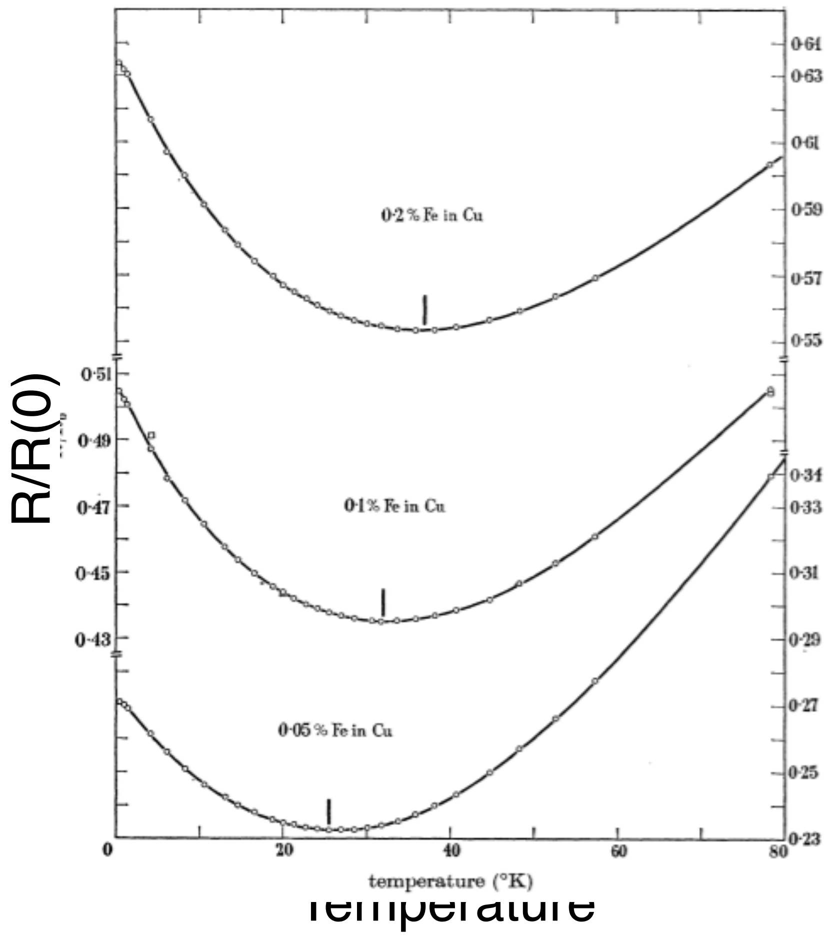
3D Graphene and Weyl Semimetals

$$\mathcal{H} = v_F \mathbf{p} \cdot \sigma$$





Kondo Effect



J. P. Franck, F. D. Manchester and Douglas L. Martin, Proc. Royal Society of London 1961

$$\rho(T) = \rho_0 + bT^5 - a\ln(T)$$

There is a crossover temperature, T_K, below which the coupling between the conduction electrons and the *dynamical* magnetic impurity grows non-perturbatively

$T<T_K$ Formation of a many-body singlet

For standard case

$$T_K = De^{-\frac{1}{J\mathbf{N(0)}}}$$

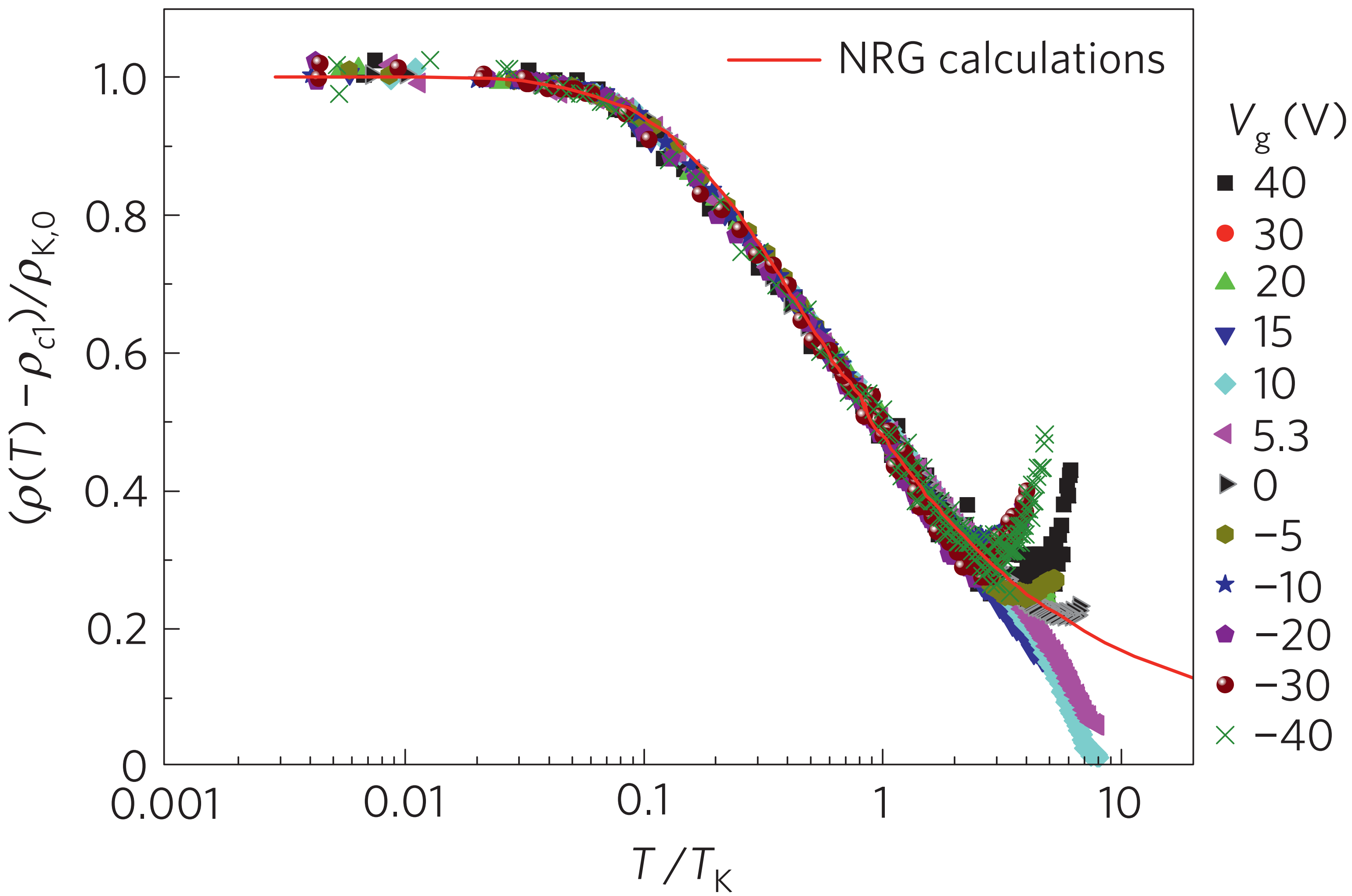
$$N(0) \propto \epsilon^{\alpha}$$

Pseudogap Kondo problem

Withoff and Fradkin(1990), Ingersent (1996), ... A.V. Balatsky et al. RMP (2006), Fritz and Vojta Reports on Progress in Physics (2013)

Experimental evidence

Graphene



Jian-Hao Chen et al. Nat. Phys. (2011)

Theoretical approach

$$H = H_{DM} + H_{\rm imp}$$

$$H_{DM} = \hbar v_F \hat{c}^\dagger_{\boldsymbol{k}\sigma}(\boldsymbol{k}\cdot\boldsymbol{\tau}_{\sigma\sigma'} - \mu)\hat{c}_{\boldsymbol{k}\sigma'}$$

$$H_{\rm imp} = J\sum_{\boldsymbol{r},\boldsymbol{R}} \hat{c}^\dagger_{\boldsymbol{r}\sigma}\boldsymbol{\tau}_{\sigma\sigma'}\hat{c}_{\boldsymbol{r}\sigma'}\cdot\boldsymbol{S}\delta(\boldsymbol{r}-\boldsymbol{R})$$

Large-N expansion

S is expressed in terms of auxiliary fermionic operators "f" satisfying the constrain

$n_f = \sum_\sigma \hat{f}^\dagger_\sigma \hat{f}_\sigma = 1$. Then

$$H_{\rm imp} = J\sum_{\boldsymbol{k},\boldsymbol{k}',\sigma} \hat{c}^\dagger_{\boldsymbol{k}\sigma}\hat{c}_{\boldsymbol{k}'\sigma'}\hat{f}^\dagger_{\sigma'}\hat{f}_\sigma$$

The interaction is decoupled via the mean-field

$$s \sim \sum_{\boldsymbol{k},\sigma}\langle \hat{f}^\dagger_\sigma \hat{c}_{\boldsymbol{k}\sigma}\rangle$$

And the constrain on n_f is enforced via a Lagrange multiplier μ_f

Determination of T_K

The field “s” and the Lagrange multiplier are obtained via the self-consistent equations:

$$\int_{-D-\mu}^{D-\mu} d\varepsilon \; \frac{n_{\mathrm{F}}(\varepsilon)(\varepsilon-\mu_f)\mathcal{N}(\varepsilon+\mu)}{(\varepsilon-\mu_f)^2+(\pi|s|^2\mathcal{N}(\varepsilon+\mu)/2)^2} = -\frac{1}{J}$$

$$\int_{-D-\mu}^{D-\mu} d\varepsilon \; \frac{n_{\mathrm{F}}(\varepsilon)|s|^2\mathcal{N}(\varepsilon+\mu)}{(\varepsilon-\mu_f)^2+(\pi|s|^2\mathcal{N}(\varepsilon+\mu)/2)^2} = 1$$

We identify T_K as the highest T
for which the two self-consistent equations admit a solution

Scalings for T_K

At the Dirac point μ = 0

3D

$$T_{\rm K} = D\frac{\sqrt{3}}{\pi}\sqrt{1-\frac{2}{\mathcal{N}(D)J}}$$

$$J_{cr} = 2/\mathcal{N}(D)$$

2D

$$T_{\rm K} = \frac{D}{\ln(4)}\left[1-\frac{1}{\mathcal{N}(D)J}\right]$$

$$J_{cr} = 1/\mathcal{N}(D)$$

Withoff and Fradkin PRL (1990), Sengupta and Baskaran PRB (2008), E. Orignac and S. Burdin PRB (2013), C. Gonzalez-Buxton and K. Ingersent PRB (1998)

Away from Dirac point μ ≠ 0

In the limit $k_{\rm B}T_{\rm K} \ll \mu \ll D$ and $J \lesssim J_{\rm c}$

3D

$$T_{\rm K} = D\exp\left[\frac{1-2/(J\mathcal{N}(D))}{2\mu^2/D^2}\right]$$

2D

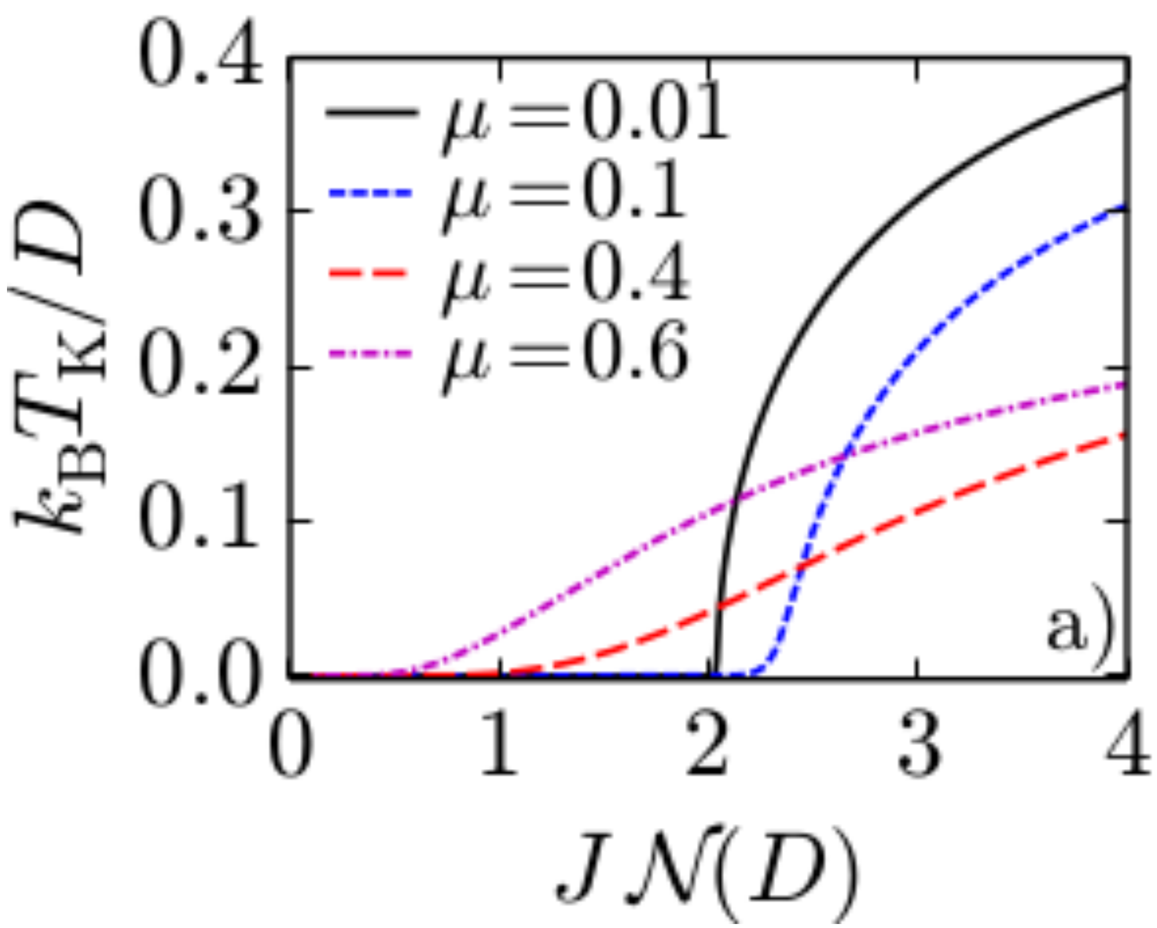
$$T_{\rm K} = \kappa(\mu)\exp\left[\frac{1-1/(\mathcal{N}(D)J)}{|\mu/D|}\right]$$

where $\kappa(\mu) = \mu^2/D$ $[\kappa(\mu) = D]$ for $\mu > 0$ $[\mu < 0]$.

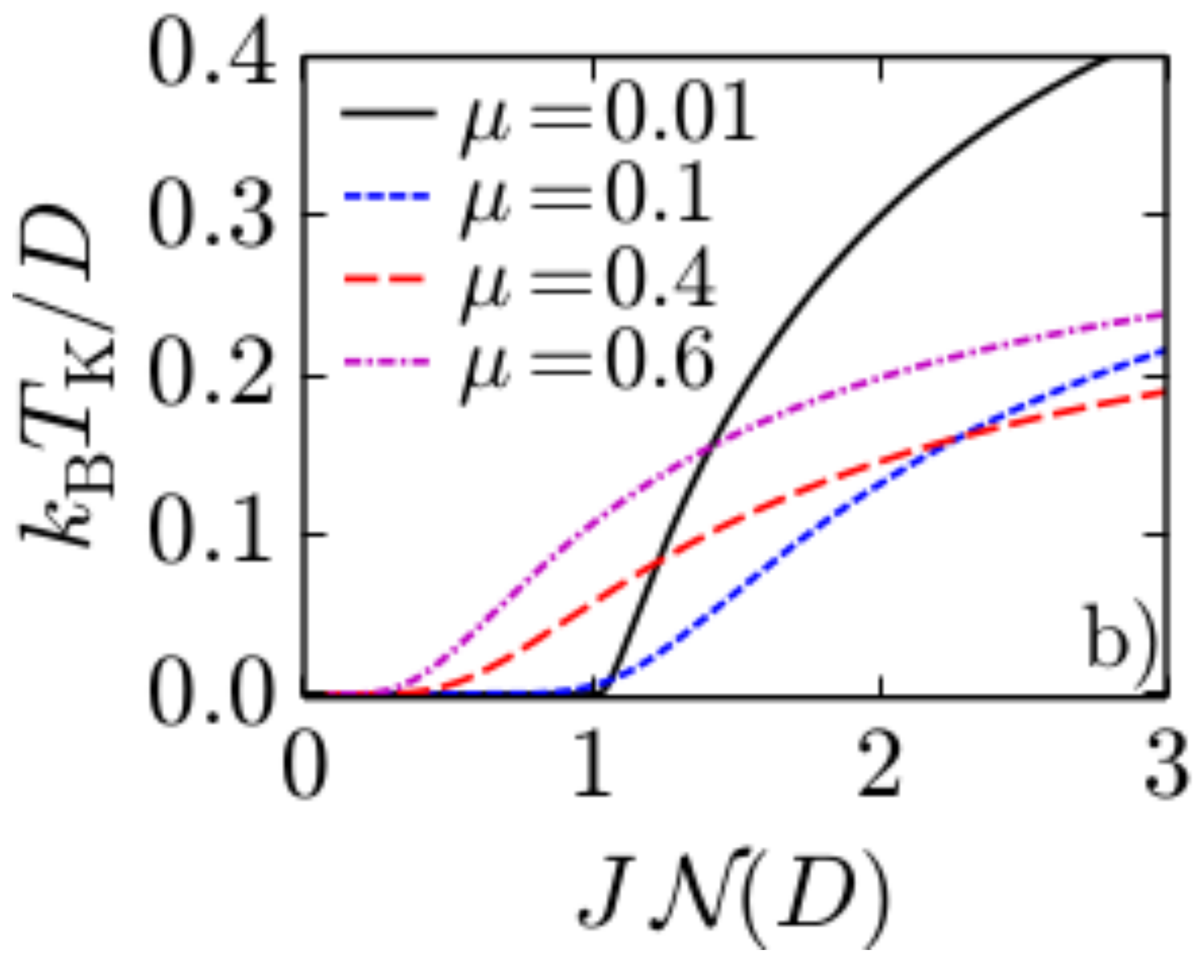
A. Principi, G. Vignale, ER, arXiv 1410.8532

Scalings for T_K: general case

3D



2D



A. Principi, G. Vignale, ER, arXiv 1410.8532

Kondo resistivity: ρ_K

In the limit T= 0

3D

$$\rho_{\rm K}(T=0) = \frac{h}{e^2}\left(\frac{32 g_{\rm s}}{3\pi^2 N_{\rm w}^2}\right)^{1/3}\frac{n_{\rm imp}}{n^{4/3}}$$

Same scaling as for the case of non magnetic long-range scatterers

AA Burkov, MD Hook, L. Balents PRB (2011)

2D

$$\rho_K = \frac{h}{e^2}\frac{4}{\pi N_{\rm w}}\frac{n_{\rm imp}}{n}$$

P.S. Cornaglia et al, PRL (2009)

At finite T

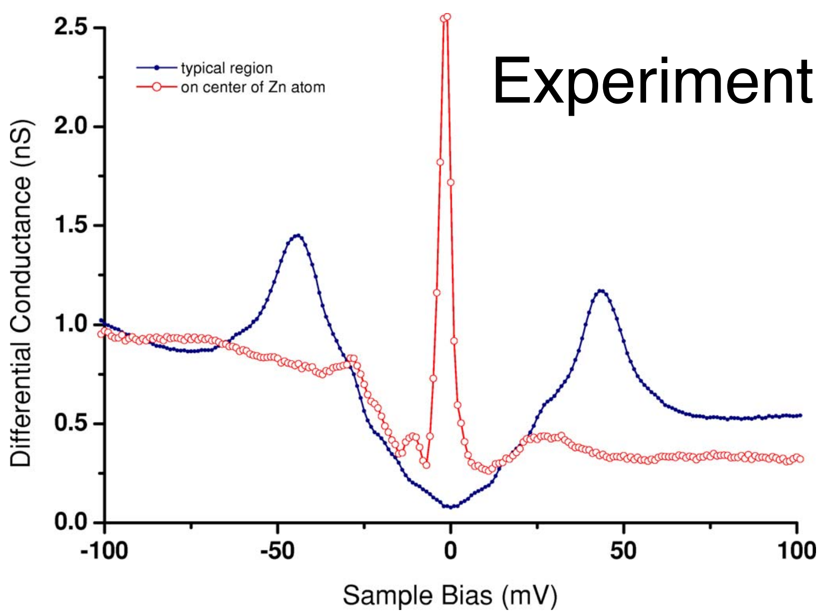
$$\rho(T) \simeq -\rho_0 \frac{\pi^2}{2}\left[J\nu(\mu)\right]^3 S(S+1)\ln\left(\frac{k_{\rm B}T}{D}\right)$$

A. Principi, G. Vignale, ER, arXiv 1410.8532

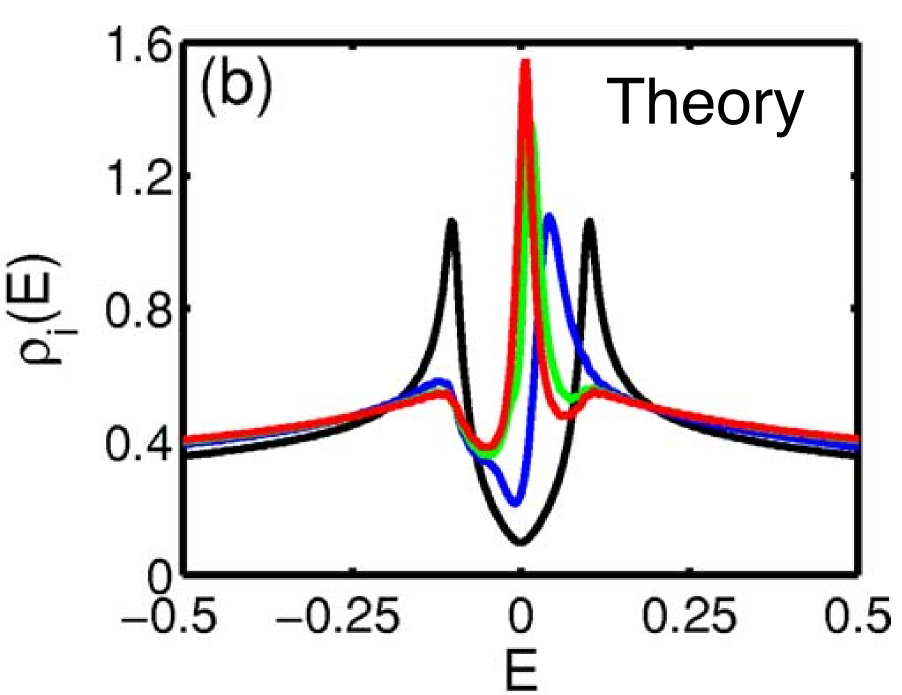
Interplay of scalar and magnetic potential

$$H_{\rm imp} = U\sum_{\boldsymbol{r},\boldsymbol{R}} \hat{c}^{\dagger}_{\boldsymbol{r}\sigma}\hat{c}_{\boldsymbol{r}\sigma}\delta(\boldsymbol{r}-\boldsymbol{R}) + J\sum_{\boldsymbol{r},\boldsymbol{R}} \hat{c}^{\dagger}_{\boldsymbol{r}\sigma}\boldsymbol{\tau}_{\sigma\sigma'}\hat{c}_{\boldsymbol{r}\sigma'}\cdot\boldsymbol{S}\delta(\boldsymbol{r}-\boldsymbol{R})$$

d-wave superconductors



Pan, S. H., E. W. Hudson, K. M. Lang, H. Eisaki, S. Uchida, and J. C. Davis, 2000, Nature (London) **403**, 746.



Buchholtz and Zwicknagl (1981), Stamp (1987), Balatsky *et al.* (1995), Salkola et al. (1996, 1997)

The scalar part of the impurity potential modifies the LDOS

It modifies T_K uniformly across the sample

T_K is uniform across the sample and well defined

If the short range scalar potential is due to *other* impurities removed from the magnetic impurity it has little consequence.

Long-range disorder

Charge impurities are a common source of disorder. However they can often be treated as short range disorder. Things are different in most Dirac materials

- Linear dispersion => vanishing DOS close to Dirac points =>poor screening of the disorder due to charge impurities

The disorder is renormalized but retains its long-range character

- Charge impurities therefore cause strong, long-range, *density* inhomogeneities close to Dirac point

- Linear dispersion

Strong, long-range, *density inhomogeneities*

$\Downarrow$

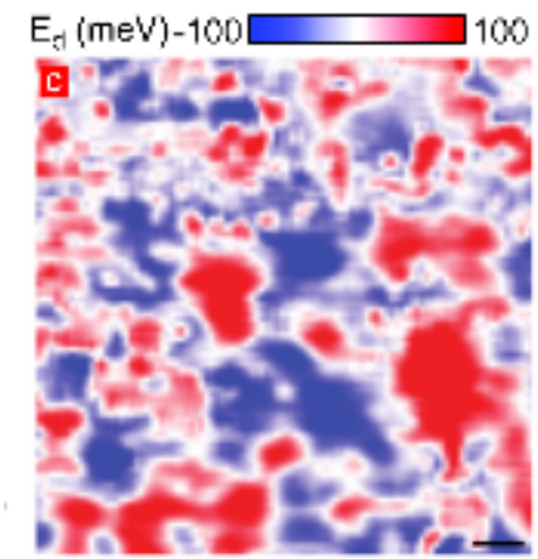
Strong, long-range, inhomogeneites of the DOS

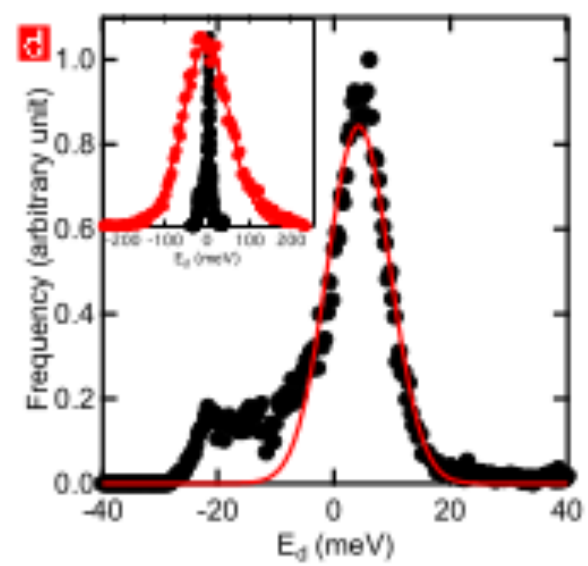
Interplay of scalar and magnetic potential: long-range scalar potential

Not important in superconductors: well screened ← Charge impurities

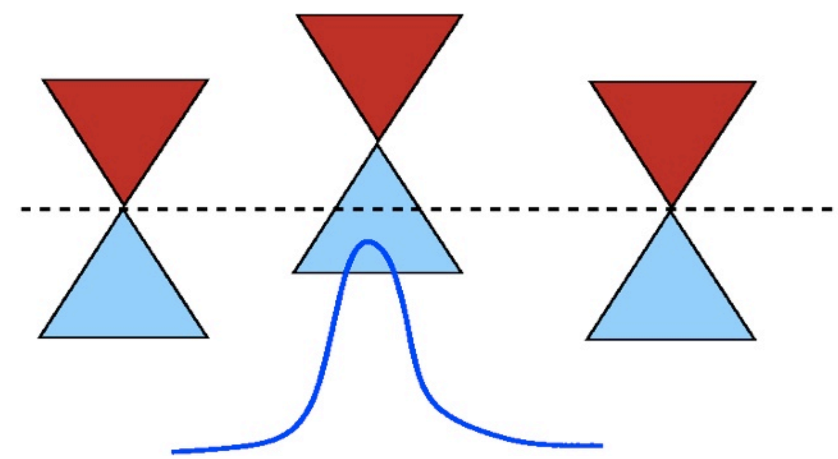
In non-superconducting Dirac materials, due to vanishing DOS they induce strong, long-range, carrier density inhomogeneities

Experiment





J. Xue et al. Nat. Mat. (2011)



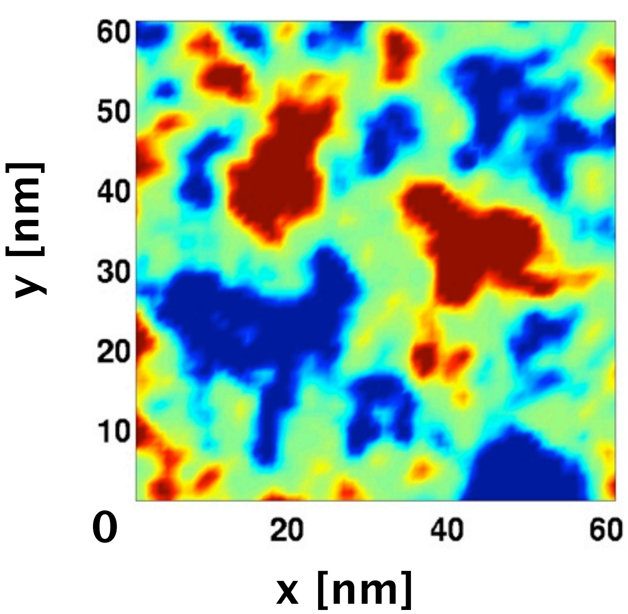
Shifts bottom of the band ↔ shift of Fermi energy δn(r)

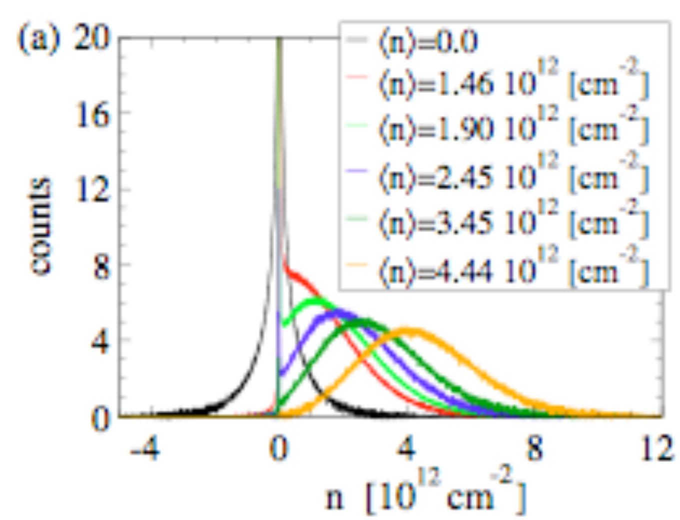
2D LDOS ~ n

3D LDOS ~ $n^{2/3}$

Fluctuations of n imply fluctuations of LDOS

Theory





ER, S. Das Sarma, PRL. (2008)

J. Martin et al. Nat Phys. (2008)
Y. Zhang et al. Nat. Phys. (2009)

Effect of long-range scalar disorder

For simplicity we assume a Gaussian distribution for the density probability

$$P_n(n) = \exp\left[-(n-\bar{n})^2/(2\sigma_n^2)\right]/(\sqrt{2\pi}\sigma_n).$$

Using the relation between T_K and μ and the fact that

in 3D: $\mu \sim n^{1/3}$ in 2D: $|\mu| \sim n^{1/2}$

Instead of a single value of T_K we have a distribution of T_K

3D

$$P^{(3\mathrm{D})}(T_\mathrm{K}) = \frac{3D^3}{8\sqrt{\pi}\sigma_\mu^3 T_\mathrm{K}}\left[\frac{(1-J_\mathrm{c}/J)^3}{\ln^5(k_\mathrm{B}T_\mathrm{K}/D)}\right]^{1/2}\left[e^{-\frac{(\mu^3-\bar{\mu}^3)^2}{2\sigma_\mu^6}} + e^{-\frac{(\mu^3+\bar{\mu}^3)^2}{2\sigma_\mu^6}}\right]$$

$$P^{(3D)} \propto \frac{1}{T_K[\ln(T_K)]^{5/2}}$$

2D

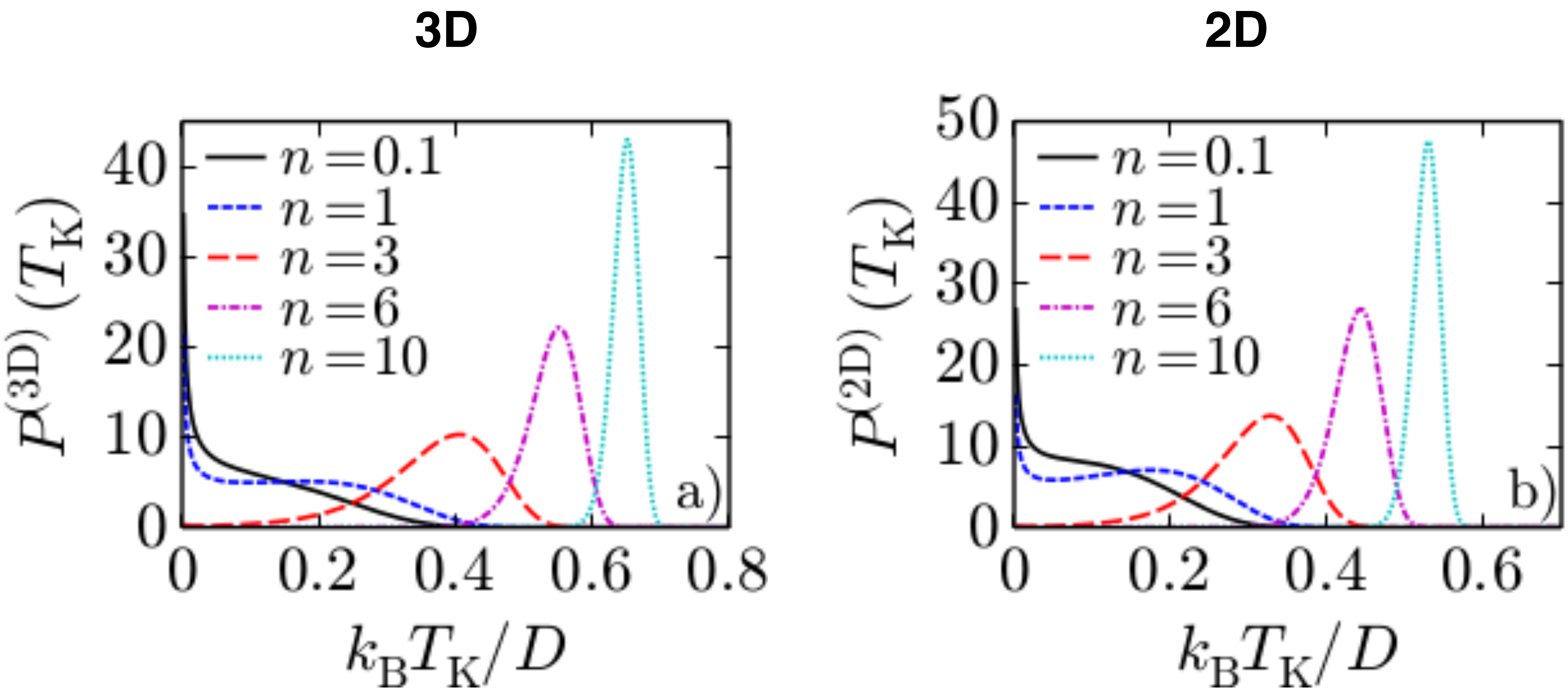
$$P^{(2\mathrm{D})}(T_\mathrm{K}) = \frac{\sqrt{2}D^2}{\sqrt{\pi}\sigma_\mu^2 T_\mathrm{K}}\frac{(1-J_\mathrm{c}/J)^2}{|\ln^3(k_\mathrm{B}T_\mathrm{K}/D)|}\left[e^{-\frac{(\mu^2-\bar{\mu}^2)^2}{2\sigma_\mu^4}} + e^{-\frac{(\mu^2+\bar{\mu}^2)^2}{2\sigma_\mu^4}}\right]$$

$$P^{(2D)} \propto \frac{1}{T_K[\ln(T_K)]^3}$$

Agrees with scaling obtain from numerical results by V.G. Miranda et al PRB (2014)

A. Principi, G. Vignale, ER, arXiv 1410.8532

$P(T_K)$



For low T_K the scaling
$1/[T_K \ln^3(T_K)]$
very similar to
$1/(T_K^{0.8})$
obtained by fitting numerical results by V.G. Miranda et al. PRB (2014)

A. Principi, G. Vignale, ER, arXiv 1410.8532

LDOS fluctuations close to MIT

Typically in materials other than Dirac materials is difficult to obtain strong, long-range fluctuations of the LDOS. A similar situation can be obtained close to a metal-insulator transition. In this case the probability distribution for the LDOS ρ is log-normal

$$P(\rho)=\frac{1}{\sqrt{4\pi u}}\frac{1}{\rho}\exp\left\{-\frac{1}{4u}\ln^2\left|\frac{\rho}{\rho_0}e^u\right|\right\}.$$

I.V. Lerner, Phys. Lett. A (1988),
B.L. Altshuler and V.N. Prigodin JETP (1987)

In this case we also get a singular distribution for T_K

$$P(T_K)=(4\pi u)^{-1/2}\frac{1}{T_K\ln(\varepsilon_F/T_K)}\exp\left\{-\frac{1}{4u}\ln^2[\rho_0 J e^{-u}\ln(\varepsilon_F/T_K)]\right\}$$

V. Dobrosavlyevic, T.R. Kirkpatrick, G. Kotliar
PRL (1992)

However:

- The conditions are difficult to achieve

- The effects are weaker than in Dirac Materials

Free carriers even for T -> 0

Considering that

$$P^{(3D)} \propto \frac{1}{T_K[\ln(T_K)]^{5/2}} \qquad P^{(2D)} \propto \frac{1}{T_K[\ln(T_K)]^{3}}$$

We see that at any is a considerable fraction of the sample for which T_k is very small

At any T, no matter how low, there is a significant fraction, n_{fr}, of carriers not **bound** to the impurities

We can obtain such fraction at temperature T by calculating the integral

$$n_{\rm fr}(T) = \int_0^T dT_{\rm K} P(T_{\rm K})$$

And we find, for T->0

3D

$$n_{\rm fr}(T) \propto |\ln(T)|^{-3/2} e^{-\bar{n}^2/(2\sigma_n^2)}$$

2D

$$n_{\rm fr}(T) \propto |\ln(T)|^{-2} e^{-\bar{n}^2/(2\sigma_n^2)}$$

Even for T -> 0 n_{fr} is significant

Non-Fermi liquid behavior

Consider the magnetic susceptibility. We have

$$\chi_{\rm m} \propto \frac{n_{\rm fr}(T)}{T}$$

And therefore we find:

3D

$$\chi_{\rm m} \propto \frac{1}{T|\ln(T)|^{3/2}}$$

2D

$$\chi_{\rm m} \propto \frac{1}{T|\ln(T)|^{2}}$$

χ

χ_m **diverges for for T->0**

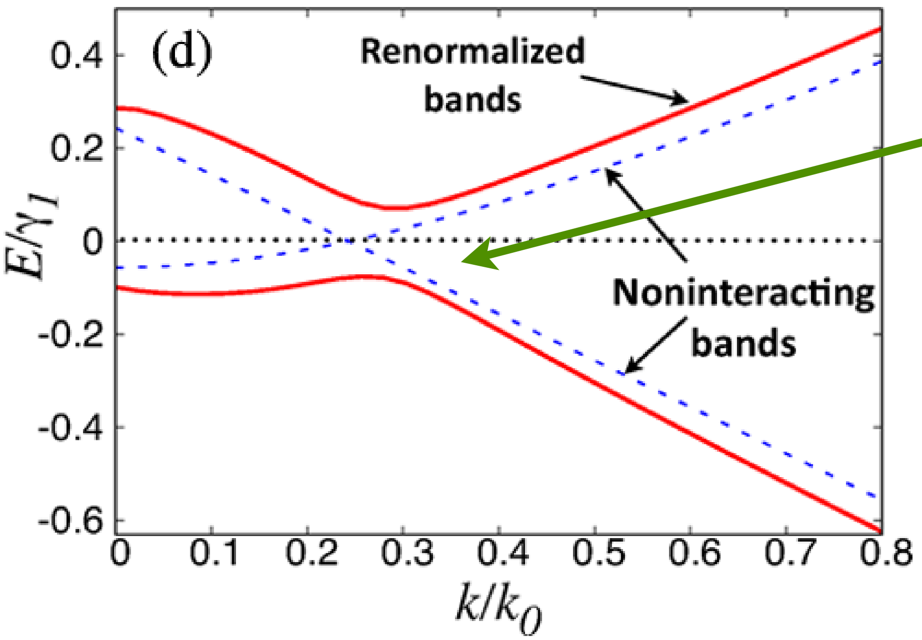
$$\boxed{\textbf{Strong Non-Fermi-Liquid Behavior}}$$

P. Nozieres (1974)

***χ_m alsp* does not follow the Curie-Weiss law (1/T) it diverges more slowly**

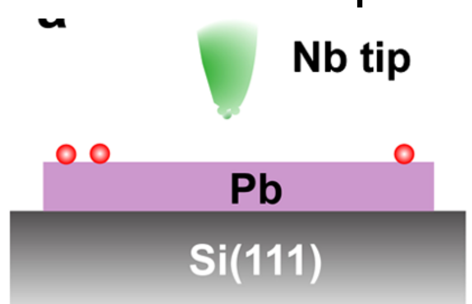
A. Principi, G. Vignale, ER, arXiv 1410.8532

Impurity-bound states in SCs with SOC: motivation

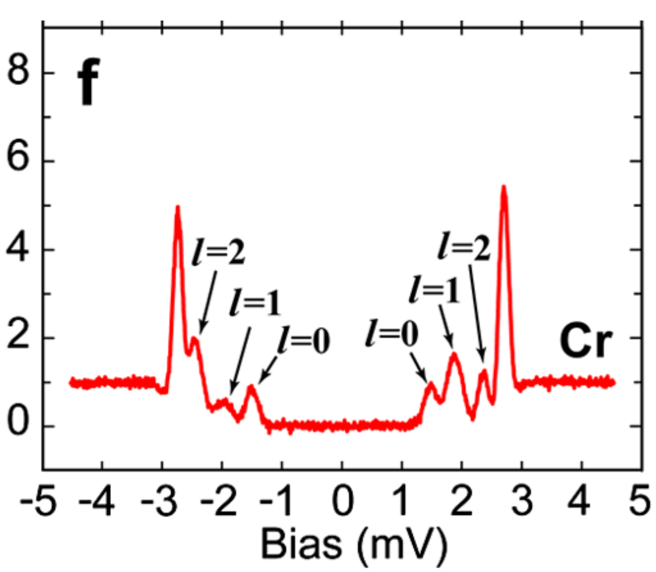


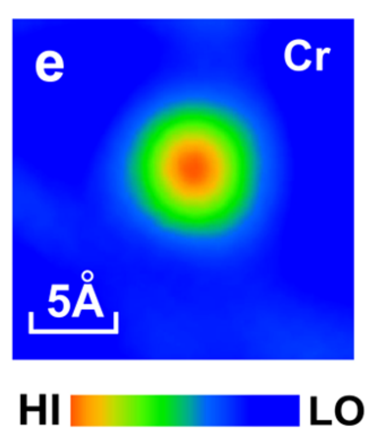
A magnetic impurity can create states with energies within the gap due to the superconducting paring. These states are spatially bound to the impurity (Yu-Shiba-Rusinov). A chain of impurities can create a band of these states.

Measured using STM for isolated impurities

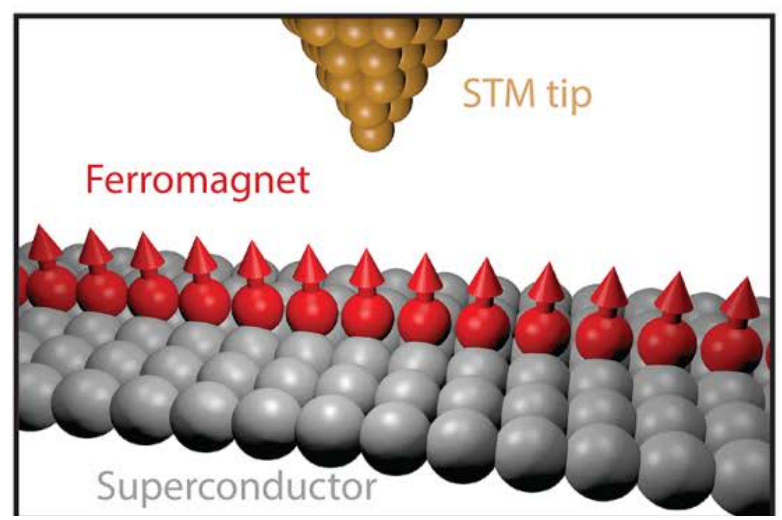


Shuai-Hua Ji et al. PRL (2008)

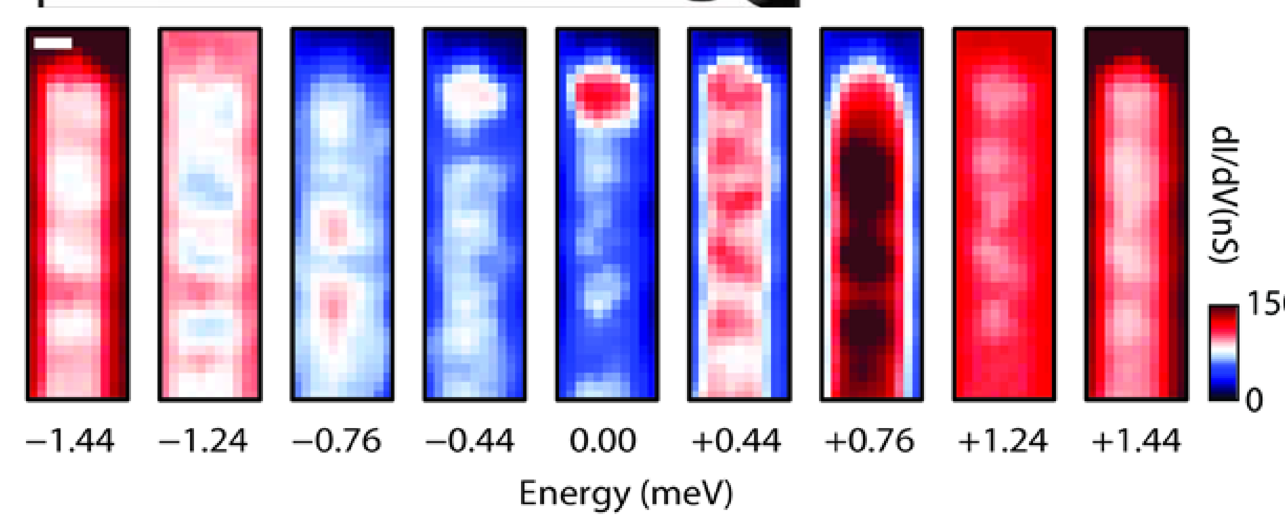




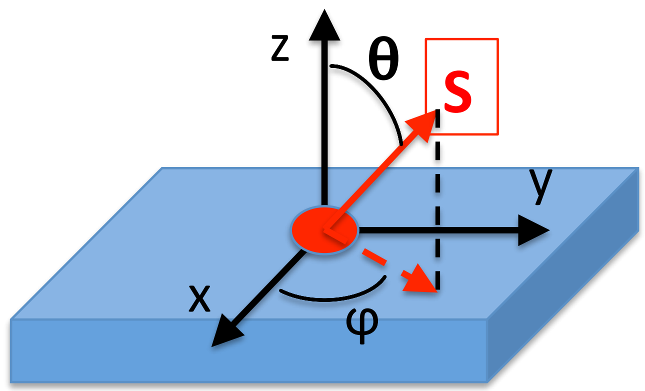
In the presence of SOC a FM chain on SC **with SOC** appears to have Majorana states at the ends



S. Nadj-Perge et al. Science (2014)



Impurity-bound states in SCs with SOC: model



$$H = H_{SC} + H_{\rm imp}$$

$$H_{SC} = \sum_{\mathbf{p}} \psi^{\dagger}_{\mathbf{p}} \left[\tau_z \otimes (\xi_{\mathbf{p}} + \alpha \mathbf{l}_{\mathbf{p}} \cdot \boldsymbol{\sigma}) + \tau_x \otimes (\Delta_0(\mathbf{p})\sigma_0 + \boldsymbol{\Delta}_{\mathbf{1}} \cdot \boldsymbol{\sigma})\right] \psi_{\mathbf{p}}$$

SOC triplet SC

$$H_{\rm imp} = \hat{U}(|\mathbf{r} - \mathbf{R}|)\tau_z \otimes \sigma_0 + \hat{J}(|\mathbf{r} - \mathbf{R}|)\tau_0 \otimes \mathbf{S} \cdot \boldsymbol{\sigma}$$

Let

$$G_{SC} \equiv [E - H_{SC}]^{-1}$$

Then the Schrodinger equation for the Hamiltonian H can be rewritten as
(F. Pientka, L. I. Glazman, and F. von Oppen, PRB (2013))

$$\psi(\mathbf{p}) - G_{SC}(E, \mathbf{p}) \int_{\mathbf{p}'} H_{\rm imp}(|\mathbf{p} - \mathbf{p}'|)\psi(\mathbf{p}') = 0.$$

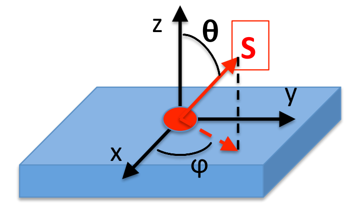
This equation admits nontrivial solutions for values of E such that

$$\det[1 - G_{SC}(E, \mathbf{p}) H_{\rm imp}(|\mathbf{p} - \mathbf{p}'|)] = 0.$$

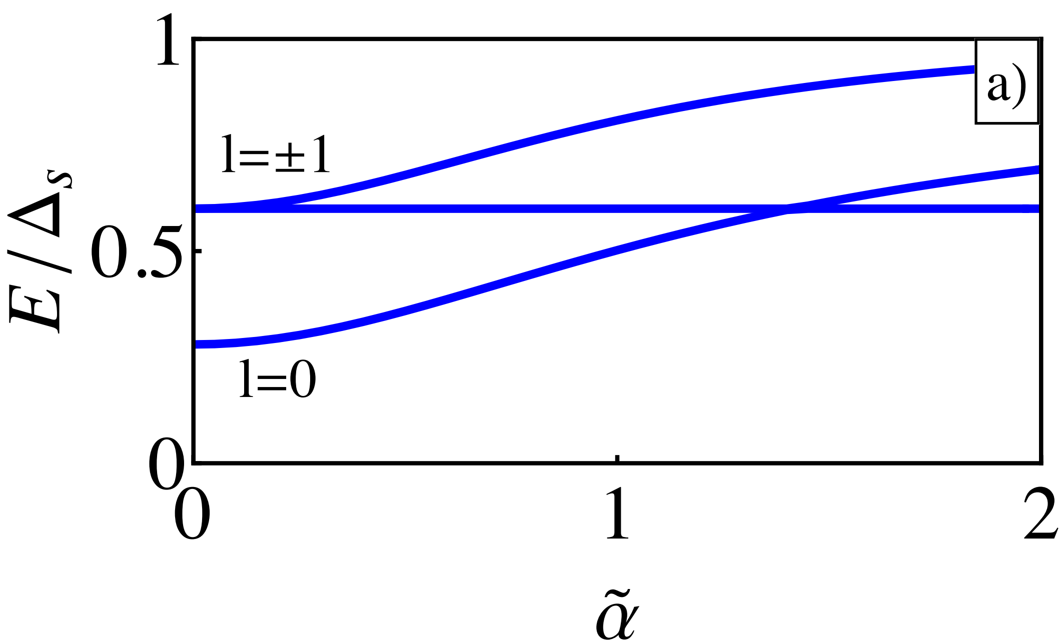
For values of |E|<Δ we have bound states (Shiba states)

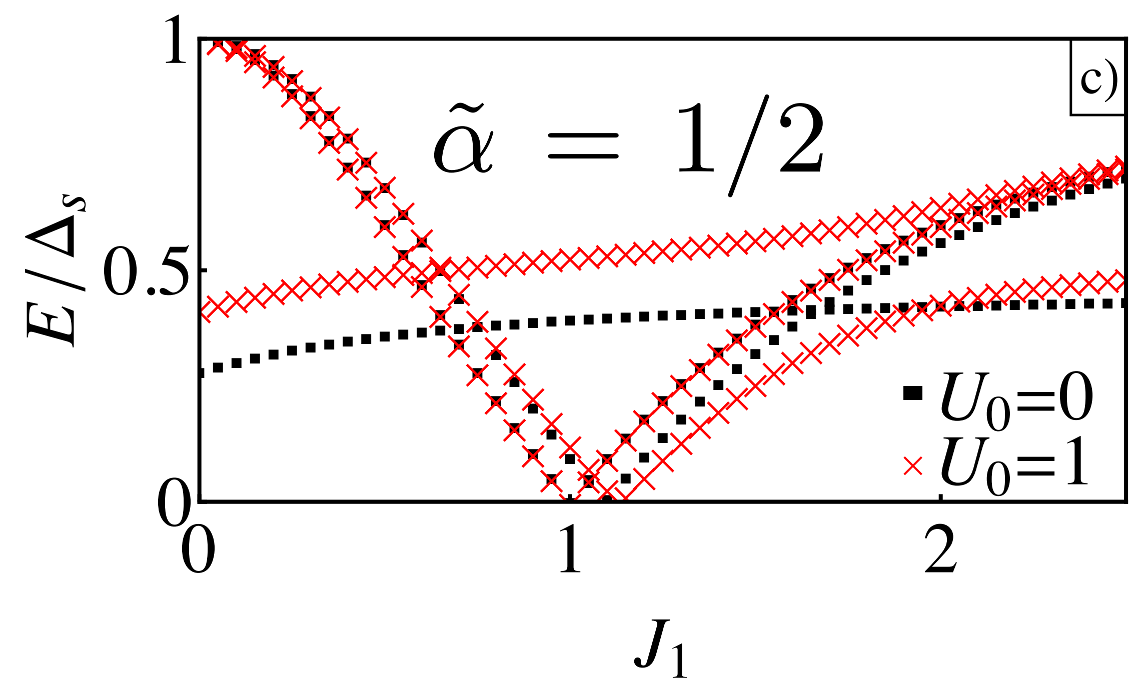
Impurity-bound states in SCs with SOC: results

s-wave superconductor



θ=0





$$\frac{|E_{l=0,1}|}{\Delta_s} = \frac{\gamma^2 - J_0^2 J_1^2 \pm \gamma^{\frac{3}{2}} \sqrt{(J_0^2 - J_1^2)^2 + (\gamma - 1)(J_0 - J_1)^4}}{\gamma^2 (1 + (J_0 - J_1)^2) + 2\gamma J_0 J_1 + J_0^2 J_1^2}$$

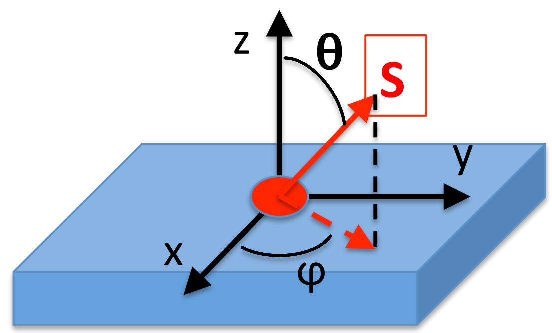
$$\gamma = 1 + \tilde{\alpha}^2$$

$$\frac{|E_{l=-1}|}{\Delta_s} = \frac{1 - J_1^2}{1 + J_1^2}$$

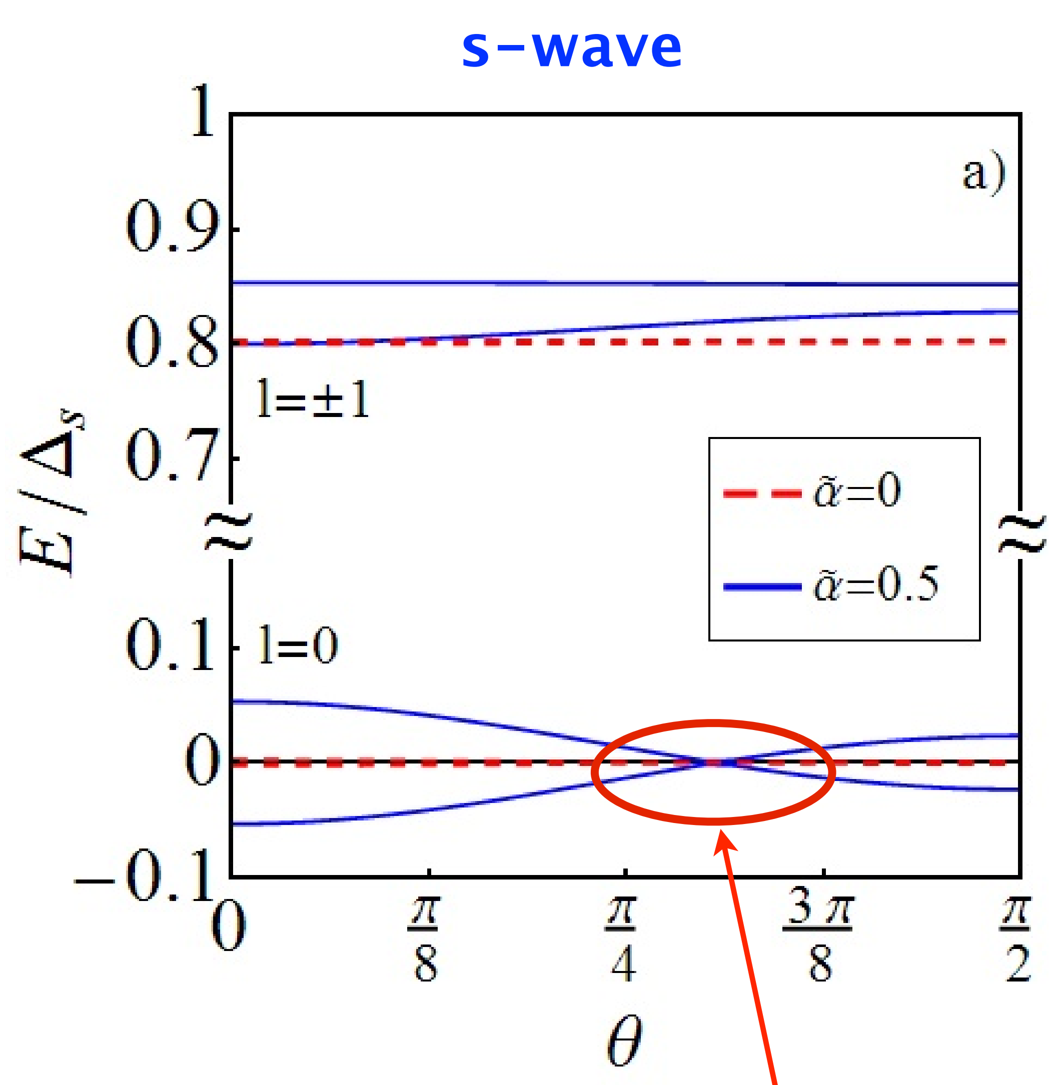
As expected SOC mixes states with different l. It also causes an interplay of U and J

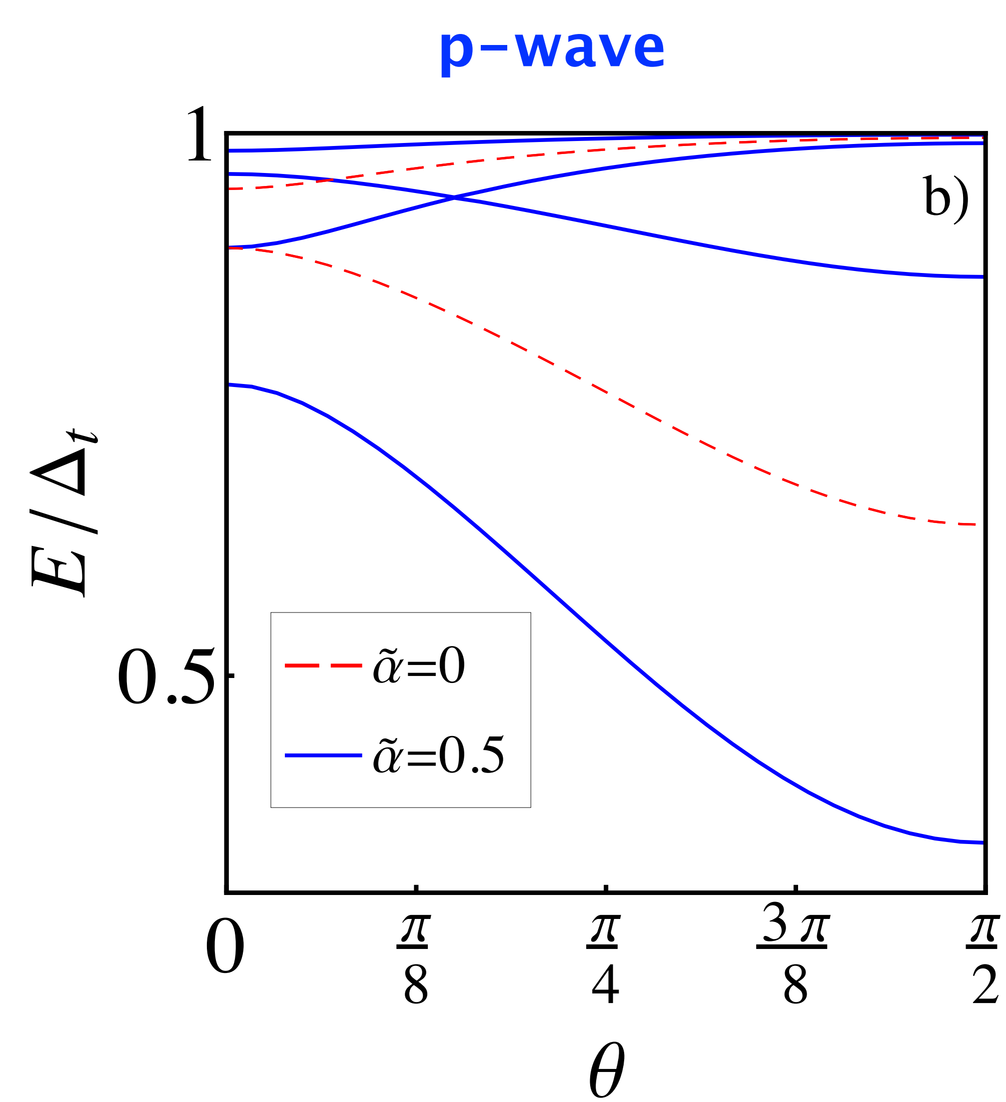
Y. Kim, J. Zhang, ER, R. Lutchyn
arXiv:1410.4558 (2014)

Impurity-bound states in SCs with SOC: results



Dependence on θ





SOC induces strong θ dependence that can be used to tune the fermion parity of the bound state

E. Rossi

Y. Kim, J. Zhang, ER, R. Lutchyn
arXiv:1410.4558 (2014)

Conclusions

- Obtained scaling of T_K and Kondo resistivity in 3D Dirac materials

$$\rho_K(T=0) \propto \frac{n_{\mathrm{imp}}}{n^{4/3}}$$

- Interplay of long-range disorder and Kondo effect in Dirac materials gives rise to a distribution of Kondo temperatures. Close to Dirac point:

$$P^{(3D)} \propto \frac{1}{T_K[\ln(T_K)]^{5/2}} \qquad P^{(2D)} \propto \frac{1}{T_K[\ln(T_K)]^3}$$

- Low T tail of $P(T_K)$ induces NFL

$$\chi_{\mathrm{m}} \propto \frac{1}{T|\ln(T)|^{3/2}} \qquad \chi_{\mathrm{m}} \propto \frac{1}{T|\ln(T)|^2}$$

- Study effect of SOC on impurity bound states in 2D superconductors

SOC strongly affects the bound states created by isolated impurities in superconductors

Can change parity of Shiba state

References

- A. Principi, G. Vignale, ER, arXiv 1410.8532 (2014)
- Y. Kim, J. Zhang, ER, R. Lutchyn, arXiv:1410.4558 (2014)

For more see:

http://physics.wm.edu/~erossi/publications.html